BLIND RECEIVERS FOR MISO COMMUNICATION SYSTEMS USING A NONLINEAR PRECODER

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ABSTRACT

In this paper, we propose two blind decoding approaches for multi-input single-output (MISO) communication systems. We first introduce a nonlinear precoding scheme that allows viewing the received signal as a Volterra-like model with the following properties: the input solely depends on the coding sequence, assumed to be known to the receiver, while the kernel is a multiway array depending on informative data and on the channel parameters. We show that such a kernel admits a PARAFAC tensor model. After estimating the kernel by using the coding sequence, the data symbols are then recovered. For this purpose, two methods are proposed. The first one directly computes the PARAFAC loading factors by means of an alternating least squares method. The second one solves the problem by means of a joint diagonalization of matrices constructed with the slices of the tensor. The performance of the proposed methods is evaluated by means of simulations.

1. INTRODUCTION

Multi-input Single Output (MISO) communication channel modelling occurs when the communication system exhibits multiple antennas and/or transmitters whereas the receiver has a single antenna (see Fig. 1).

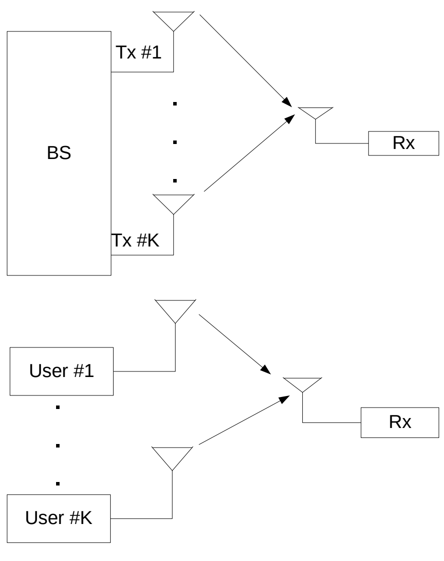

Figure 1: Multi Input Single Output communication systems.

In both cases several space-time block processing coding and modulation schemes (in a distributed or cooperative way in the second case) have been proposed in the literature. For scenarios in which there is perfect channel state information (CSI), several linear precoding systems have been proposed (see [1] and references therein). However, in practice the CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback [2]. The derivation of robust coding methods with few or no knowledge on the transmission channel is then a topic of particular interest.

In MISO communication channels, the propagation scenario can be viewed as a highly underdetermined mixture of sources having more sources than sensors. Systems with one single output sensor have received considerably less attention (see [3] and references therein).

In this paper, we introduce a nonlinear precoding scheme where the CSI is not required. From such a scheme, we derive blind decoding approaches for a MISO communication system. The proposed nonlinear precoding scheme gives rise to a homogeneous Volterra-like input-output equation whose inputs depend on the coding sequence whereas the kernel depends on the informative symbols and on the channel parameters.

It is now well known that Volterra kernels of order higher than two can be viewed as tensors or multiway array. We show that the kernel of the resulting Volterra-like model admits a PARAFAC model. PARAFAC (PARallel FACtors analysis) [4] is certainly the most famous tensor model proposed in the literature. In the last decade, several PARAFAC or more generally tensor based signal processing methods have been proposed in the literature devoted to communications [5–9]. Most of them make use of the spatial diversity induced by multiple receive antennas. The contribution of this paper is to provide a nonlinear precoding scheme at the transmitter end and appropriate algorithms for blind decoding. The first decoding approach is based on the alternating least squares algorithm whereas in the second one the problem is solved by means of a joint diagonalization of a set of matrices constructed from tensor slices. The paper is organized as follows. In section 2, we explain the nonlinear encoding scheme and deduce the overall system model. In section 3, two-steps decoding schemes are derived. In section 4, the performance of the derived methods are evaluated by means of simulations before concluding the paper in section 5.

Notations: Vectors are written as boldface lower-case letters $(\mathbf{a},\mathbf{b},\cdots)$ and matrices as boldface capitals $(\mathbf{A},\mathbf{B},\cdots)$. Tensors are written using calligraphic letters $\mathscr{X}$. $\mathbf{A}_{i.}$ and $\mathbf{A}_{.j}$ denote respectively the ith row and the jth column of the $I \times J$ matrix $\mathbf{A}$. $\mathbf{A}^T$ stands for the transpose of $\mathbf{A}$ whereas $\mathbf{A}^H$ stands for its complex conjugate. $\mathbf{A}^\dagger$ stands for the matrix pseudo-inverse. $diag(.)$ is the operator that forms a diagonal matrix from its vector argument whereas $vec(.)$ forms a vector by stacking the columns of its matrix argument. For $\mathbf{X} \in \mathbb{C}^{I \times R}$ and $\mathbf{Y} \in \mathbb{C}^{J \times R}$, the Khatri-Rao product, denoted by $\odot$, is defined as follows:

$$\mathbf{X} \odot \mathbf{Y} = \begin{pmatrix} \mathbf{Y} diag(\mathbf{X}_{1.}) \\ \vdots \\ \mathbf{Y} diag(\mathbf{X}_{I.}) \end{pmatrix} \in \mathbb{C}^{IJ \times R} \quad (1)$$

It can also be viewed as a column-wise Kronecker product.

$$\mathbf{X} \odot \mathbf{Y} = \begin{pmatrix} \mathbf{X}_{.1} \otimes \mathbf{Y}_{.1} & \cdots & \mathbf{X}_{.R} \otimes \mathbf{Y}_{.R} \end{pmatrix} \in \mathbb{C}^{IJ \times R}, \quad (2)$$

$\otimes$ denoting the Kronecker product.

2. SYSTEM MODEL

The considered communication system has K multiple transmitters or antennas. Each user transmits digital signals at the same time and using the same bandwidth. The output at the receiver is then a superposition of K signal waveforms. For each user, the QM-length symbol stream is first parsed into $M \times 1$ symbol vectors $\mathbf{s}_q^{(k)} = \begin{pmatrix} s_{1,q}^{(k)} & \cdots & s_{M,q}^{(k)} \end{pmatrix}^T$, $q = 1, \cdots, Q$. The nonlinear precoding considered herein is a two-stage one. First, each of the symbol vectors $\mathbf{s}_q^{(k)}$ is linearly precoded by an $N \times M$ matrix $\mathbf{A}$. We get $\mathbf{b}_q^{(k)} = \mathbf{A}\mathbf{s}_q^{(k)}$. Note that the linear coding matrix is the same for all the users. Then, the codewords $\mathbf{c}_q^{(k)}$ to be transmitted are obtained through a nonlinear mapping $f(.)$:

$$\mathbf{c}_q^{(k)} = f(\mathbf{b}_q^{(k)}) = f(\mathbf{A}\mathbf{s}_q^{(k)}). \quad (3)$$

The codewords are modulated by a pulse-shape filter $g_k(t)$ so that the baseband signal $x_{k,q}(t)$ transmitted by the kth user is given by

$$x_{k,q}(t) = \sum_{n=1}^{N} c_{n,q}^{(k)} g_k\left(t - (n-1)T\right),$$

T being an appropriately chosen fraction of the symbol period T_s.
We assume that each of the signals $x_{k,q}(t)$, $k = 1, \cdots, K$, is received via a single path characterized by a fading-factor α_k and a delay τ_k that holds propagation delay and asynchronism. In baseband, the received signal $y_q(t)$ is then given by:

$$y_q(t) = \sum_{k=1}^{K} \alpha_k x_{k,q}(t - \tau_k) + w_q(t),$$

$w_q(t)$ denoting the additive noise. By sampling at The discrete-time baseband equivalent model for the received data is then given by:

$$y_{n,q} = y_q(t)|_{t=(n-1)T} = \sum_{k=1}^{K} h_{k,q} \mathbf{c}_{n,q}^{(k)} + w_{n,q}$$

with $h_{k,q} = \alpha_k g_k\left(t - (n-1)T - \tau_k\right)|_{t=(n-1)T}$ assumed to be quasi-static, i.e. constant during the transmission of the qth data block.

In the sequel, we consider that the nonlinear function $f(.)$ involved in the encoding process (3) is a pth degree monomial. Therefore, elementwise the received data can be written as follows:

$$\begin{aligned} y_{n,q} &= \sum_{k=1}^{K} h_{k,q} c_{n,q}^{(k)} + w_{n,q} = \sum_{k=1}^{K} h_{k,q} f\Big(\sum_{m=1}^{M} a_{n,m} s_{m,q}^{(k)}\Big) + w_{n,q} \\ &= \sum_{k=1}^{K} h_{k,q} \left(\sum_{m=1}^{M} a_{n,m} s_{m,q}^{(k)}\right)^p + w_{n,q} \\ &= \sum_{k=1}^{K} \sum_{m_1=1}^{M} \cdots \sum_{m_p=1}^{M} h_{k,q} \prod_{j=1}^{p} a_{n,m_j} s_{m_j,q}^{(k)} + w_{n,q}. \end{aligned} \quad (4)$$

The aim of our study is to derive estimators of the data symbols $s_{m,q}^{(k)}$ solely from the received data $y_{n,q}$. We assume that the linear precoding matrix $\mathbf{A}$ is known to the receiver. By defining

$$\beta_{m_1,\cdots,m_p,q} = \sum_{k=1}^{K} h_{k,q} \prod_{j=1}^{p} s_{m_j,q}^{(k)}, \quad (5)$$

we can rewrite (4) as:

$$y_{n,q} = \sum_{m_1=1}^{M} \cdots \sum_{m_p=1}^{M} \beta_{m_1,\cdots,m_p,q} \prod_{j=1}^{p} a_{n,m_j}. \quad (6)$$

We can note from (6) that the received signal is linear in the unknown $\beta_{m_1,\cdots,m_p,q}$ but nonlinear in the coding matrix entries. In fact, in a system theory point-of-view, Eq. (6) can be viewed as the input-output equation of a pth-order homogeneous Volterra model [10], where $\beta_{m_1,\cdots,m_p,q}$ and $a_{n,m}$ represent respectively the Volterra kernel and the input sequence. Moreover, the structure of the kernel (5) looks like that of a parallel cascade Wiener model (see [11]). Therefore, in the sequel, we derive two-stage receivers. The first step consists in estimating the parameters $\beta_{m_1,\cdots,m_p,q}$ in the least squares sense whereas the second one make use of the algebraic structure of the estimated parameters.

In the sequel, we restrict our study to the third-order case, $p = 3$.

3. BLIND RECEIVERS

The parameters $\beta_{m_1,m_2,m_3,q}$ can be viewed as entries of a symmetric tensor. Indeed, for any permutation $\pi(.)$ of the indices (m_1,m_2,m_3), we have $\beta_{p_1,p_2,p_3,q} = \beta_{m_1,m_2,m_3,q}$ with $(p_1,p_2,p_3) = \pi(m_1,m_2,m_3)$. We can then rewrite (6), in the noiseless case, as

$$y_{n,q} = \sum_{m_1=1}^{M} \sum_{m_2=m_1}^{M} \sum_{m_3=m_2}^{M} \tilde{\beta}_{m_1,m_2,m_3,q} \prod_{j=1}^{3} a_{n,m_j}. \quad (7)$$

where

$$\tilde{\beta}_{m_1,m_2,m_3,q} = \begin{cases} \beta_{m_1,m_2,m_3,q} & \text{if} \quad m_1 = m_2 = m_3 \\ 3\beta_{m_1,m_2,m_3,q} & \quad m_1 = m_2 \neq m_3 \\ 3\beta_{m_1,m_2,m_3,q} & \quad m_1 = m_3 \neq m_2 \\ 3\beta_{m_1,m_2,m_3,q} & \quad m_2 = m_3 \neq m_1 \\ 6\beta_{m_1,m_2,m_3,q} & \quad m_1 \neq m_2 \neq m_3 \end{cases}$$

In matrix form, Eq. (7) can be written as follows:

$$\mathbf{y}_q = \begin{pmatrix} y_{1,q} & \cdots & y_{N,q} \end{pmatrix}^T = \mathbf{\Phi}\theta_q, \quad (8)$$

where θ_q is a $\bar{Q} \times 1$ vector containing the parameters $\tilde{\beta}_{m_1,m_2,m_3,q}$ to be estimated, $\mathbf{\Phi}$ is an $N \times \bar{Q}$ matrix defined as $\mathbf{\Phi} = \mathbf{\Psi}_A \mathbf{\Omega}$, with $\mathbf{\Psi}_A \begin{pmatrix} \mathbf{A}_{1.} \otimes \mathbf{A}_{1.} \otimes \mathbf{A}_{1.} \\ \vdots \\ \mathbf{A}_{N.} \otimes \mathbf{A}_{N.} \otimes \mathbf{A}_{N.} \end{pmatrix}$, $\mathbf{\Omega}$ is a $M^3 \times \bar{Q}$ column selection matrix, and $\bar{Q} = (M+2)(M+1)M/6$. The least square solution of (8) is given by:

$$\hat{\theta}_q = \mathbf{\Phi}^{\dagger} \mathbf{y}_q \tag{9}$$

provided $\mathbf{\Phi}$ is full column rank. Therefore, the most important criterion for designing the coding matrix $\mathbf{A}$ is to ensure that $\mathbf{\Phi}$ be full column rank. The design of the encoder is then decoupled from the channel knowledge. However, in order to improve the quality of the estimates in a noisy framework it could be necessary to increase N.

Once the parameters $\tilde{\beta}_{m1,m_2,m_3,q}$ have been estimated, we can deduce $\beta_{m1,m2,m3,q}$. Therefore, we will estimate the informative symbols from the estimated parameters $\beta_{m1,m2,m3,q}$, which can be viewed as the entries of a third-order tensor. In the sequel, we remove the index q since the decoding process is per-block.

3.1 PARAFAC tensor model

Let us denote by $\mathscr{B}$ the $M \times M \times M$ third-order symmetric tensor with β_{m_1,m_2,m_3} as entries. From Eq. (5), we can deduce that $\mathscr{B}$ admits a PARAFAC model [4] with $\mathbf{S}$ and $\mathbf{S}diag(\mathbf{h})$ as factor matrices. Using the Kruskal operator [12, 13], we get:

$$\mathscr{B} = [\mathbf{S}, \mathbf{S}, \mathbf{S}diag(\mathbf{h})]$$

with

$$\mathbf{h} = \begin{pmatrix} h_{1,q} & \cdots & h_{K,q} \end{pmatrix}^T$$

and

$$\mathbf{S} = \begin{pmatrix} s_{1,q}^{(1)} & \cdots & s_{1,q}^{(K)} \\ \vdots & \ddots & \vdots \\ s_{M,q}^{(1)} & \cdots & s_{M,q}^{(K)} \end{pmatrix}$$

the matrix of the data symbols assumed to be full column rank, which implies $M \geq K$.

From the sufficient condition stated by Kruskal [12], we can deduce that, the factor matrices are essentially unique, i.e. unique up to column permutation and scaling, if $k_{\mathbf{S}} \geq \frac{2}{3}(K+2)$, where $k_{\mathbf{S}}$ denotes the Kruskal-rank of $\mathbf{S}$. It is also called k-rank and is defined as the greatest integer $k_{\mathbf{S}}$ such that any set of $k_{\mathbf{S}_t}$ columns of $\mathbf{S}$ is independent. Moreover, since the columns of $\mathbf{S}$ are associated with independent users, for $M \geq K$, $\mathbf{S}$ is full column rank with a high probability. As a consequence, the above inequality is always satisfied. Hence, the factor matrices can be obtained up to a scaling factor. The scaling ambiguity can be removed by considering differential modulation or by setting the first row of $\mathbf{S}$ equals to one.

Before deriving the estimation algorithm for fitting the PARAFAC model, we define the following matrix representations of the tensor. The slices of $\mathscr{B}$ are given by

$$\begin{aligned} \mathbf{B}_{m..} = \mathbf{B}_{.m.} = \mathbf{B}_{..m} &= \begin{pmatrix} \beta_{1,1,m} & \cdots & \beta_{1,M,m} \\ \vdots & \ddots & \vdots \\ \beta_{M,1,m} & \cdots & \beta_{M,M,m} \end{pmatrix} \\ &= \mathbf{S}diag(\mathbf{S}_{m.})diag(\mathbf{h})\mathbf{S}^T. \end{aligned} \tag{10}$$

By concatenating these slices, we get the unfolding matrix

$$\begin{aligned} \mathbf{B} &= \begin{pmatrix} \mathbf{B}_{1..} \\ \vdots \\ \mathbf{B}_{M..} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{1..} \\ \vdots \\ \mathbf{B}_{M..} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{1..} \\ \vdots \\ \mathbf{B}_{M..} \end{pmatrix} \\ &= (\mathbf{S} \odot \mathbf{S})\, diag(\mathbf{h}_t)\mathbf{S}^T. \end{aligned} \tag{11}$$

For fitting the parameters of the PARAFAC model, we make use of an Alternating least squares algorithm. For this purpose, we define $\mathbf{A}_1 = \mathbf{S}$, $\mathbf{A}_2 = \mathbf{S}$ and $\mathbf{A}_3 = \mathbf{S}diag(\mathbf{h})$, so that we can rewrite the unfolding matrix as follows:

$$\mathbf{B} = (\mathbf{A}_1 \odot \mathbf{A}_2)\, \mathbf{A}_3^T = (\mathbf{A}_2 \odot \mathbf{A}_3)\, \mathbf{A}_1^T = (\mathbf{A}_3 \odot \mathbf{A}_1)\, \mathbf{A}_2^T.$$

The alternating least squares algorithm consists in alternating minimization of the cost functions

$$\begin{aligned} J_1 &= \left\| \mathbf{B} - (\mathbf{A}_2 \odot \mathbf{A}_3)\, \mathbf{A}_1^T \right\|_F^2, \\ J_2 &= \left\| \mathbf{B} - (\mathbf{A}_3 \odot \mathbf{A}_1)\, \mathbf{A}_2^T \right\|_F^2, \\ J_3 &= \left\| \mathbf{B} - (\mathbf{A}_1 \odot \mathbf{A}_2)\, \mathbf{A}_3^T \right\|_F^2. \end{aligned}$$

For each cost functions, given the two matrices involved in the Khatri-Rao product, the least squares solutions are respectively:

$$\begin{aligned} \hat{\mathbf{A}}_1^T &= (\mathbf{A}_2 \odot \mathbf{A}_3)^{\dagger}\, \mathbf{B}, \\ \hat{\mathbf{A}}_2^T &= (\mathbf{A}_3 \odot \mathbf{A}_1)^{\dagger}\, \mathbf{B}, \\ \hat{\mathbf{A}}_3^T &= (\mathbf{A}_1 \odot \mathbf{A}_2)^{\dagger}\, \mathbf{B}. \end{aligned}$$

After convergence, assuming that $\mathbf{S}$ has 1s as entries of its first row, its estimate is given by:

$$\hat{\mathbf{S}} = \frac{1}{3}\left(\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 diag(\mathbf{a})^{-1}\right)$$

with $\mathbf{a}$ the first row of $\mathbf{A}_3$. We can summarize the ALS estimation method as follows:

1. Initialize $\hat{\mathbf{A}}_1^{(i)}$ and $\hat{\mathbf{A}}_2^{(i)}$, $i = 0$.
2. Increment $i = i + 1$.
3. Compute $\hat{\mathbf{A}}_3^{(i)} = \left(\left(\mathbf{A}_1^{(i-1)} \odot \mathbf{A}_2^{(i-1)} \right)^{\dagger} \mathbf{B} \right)^T$.
4. Compute $\hat{\mathbf{A}}_1^{(i)} = \left(\left(\mathbf{A}_2^{(i-1)} \odot \mathbf{A}_3^{(i)} \right)^{\dagger} \mathbf{B} \right)^T$.
5. Compute $\hat{\mathbf{A}}_2^{(i)} = \left(\left(\mathbf{A}_3^{(i)} \odot \mathbf{A}_1^{(i)} \right)^{\dagger} \mathbf{B} \right)^T$.
6. Go to step 2 until a stoping criterion is reached.
7. Compute $\hat{\mathbf{S}} = \frac{1}{3}\left(\mathbf{A}_1^{(i)} + \mathbf{A}_2^{(i)} + \mathbf{A}_3^{(i)} diag(\mathbf{a}^{(i)})^{-1} \right)$, where $\mathbf{a}^{(i)}$ denotes the first row of $\mathbf{A}_3^{(i)}$.

3.2 Joint diagonalization approach

Assuming that $\mathbf{S}$ is full column rank, we can deduce that $\mathbf{S} \odot \mathbf{S}$ is also full column rank [14]. Obviously, $diag(\mathbf{h})\mathbf{S}^T$ is full row rank, and therefore $rank(\mathbf{B}) = K$, i.e. $\mathbf{B}$ is a rank deficient matrix if $M > K$.

Let us now consider the reduced singular value decomposition (SVD) of $\mathbf{B}$:

$$\mathbf{B} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \tag{12}$$

the column-orthonormal matrices $\mathbf{U}$ and $\mathbf{V}$, with respective dimensions $M^2 \times K$ and $M \times K$, containing the left and right singular vectors of $\mathbf{B}$ respectively, whereas the $K \times K$ diagonal matrix $\mathbf{\Sigma}$ is formed with the nonzero singular values of $\mathbf{B}$.

From equations (11) and (12), and the fact that $rank(\mathbf{B}^T) = K$, we deduce that $\mathbf{V}$ and $\mathbf{S}$ span the same column space. So, there exists a nonsingular matrix $\mathbf{F}$, with dimensions $K \times K$, such that

$$\mathbf{S} = \mathbf{V}\mathbf{F}. \tag{13}$$

We can then rewrite the tensor slices as follows:

$$\mathbf{B}_{..m} = \mathbf{V}\mathbf{F} diag(\mathbf{S}_{m.}) diag(\mathbf{h}) \mathbf{F}^T \mathbf{V}^T.$$

Now, let us define the following symmetric matrices:

$$\mathbf{G}_m = \mathbf{V}^T \mathbf{B}_{..m} \mathbf{V} = \mathbf{F} diag(\mathbf{S}_{m.}) diag(\mathbf{h}) \mathbf{F}^T, \tag{14}$$

with $m = 1, \cdots, M$. We can conclude that $\mathbf{F}$ jointly diagonalizes the matrices $\mathbf{G}_m$, $m = 1, \cdots, M$. Therefore $\mathbf{F}$ can be obtained by solving a joint diagonalization problem using one of the joint diagonalization algorithms proposed in the literature ([15] for example). Then, $\mathbf{S}$ is estimated using (13). The decoding process is summarized as follows:

1. Compute the matrix $\mathbf{V}$ of the K right singular vectors of $\mathbf{B}$.
2. Construct the set of matrices $\mathbf{G}_m$, $m = 1, \cdots, M$ as follows $\mathbf{G}_m = \mathbf{V}^T \mathbf{B}_{..m} \mathbf{V}$.
3. Find the $K \times K$ matrix $\mathbf{F}$ that jointly diagonalizes the matrices $\mathbf{G}$.
4. Compute the data matrix as $\hat{\mathbf{S}} = \mathbf{V}\mathbf{F}$.

4. SIMULATION RESULTS

In this section, we give some simulation results. The simulated communication system was characterized by the following parameters: $K = M = 3$. The data sequences was BPSK ones. Both channel parameters and encoding matrix were driven from a uniform distribution. The results presented below are averaged values over 100 Monte Carlo trials. The decoding performance is evaluated in terms of bit-error-rate (BER). The joint diagonalization method used in the second decoding approach is the FFDIAG method [15].

For each decoding method, in Fig. 2 and 3 we plot the BER according to the signal-to-noise ratio (SNR).

In general, the proposed decoding methods give good results. Significant improvements are obtained by increasing the number N of rows for the encoding matrix $\mathbf{A}$. That is an expected result since by increasing the number of rows for the encoding matrix, the least squares estimation of the data tensor is improved. The improvement is particularly significant for SNR values higher than 2 dB.

In figures 4, 5, and 6 we compare the two decoding methods for different values of N. We obtain comparable results with both methods. The joint diagonalization approach gives slightly better results. Note that the ALS-PARAFAC were randomly initialized. We considered 10 different initialization and then that giving the best results was selected. The algorithm were stopped after 100 iterations. For these simulations the joint diagonalization approach seems to have more desirable features.

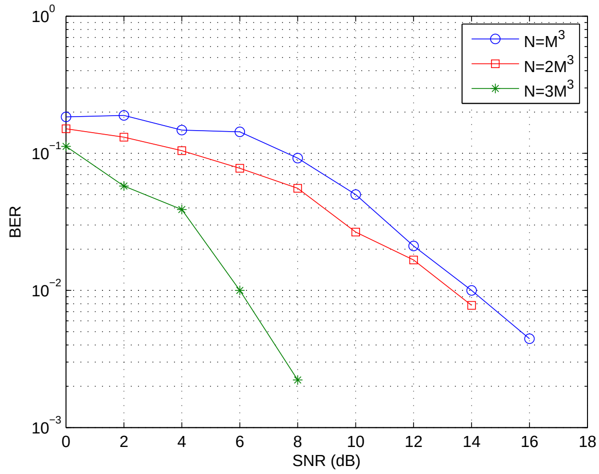

Figure 2: Performance evaluation with different number of rows for the encoding matrix (ALS-PARAFAC case).

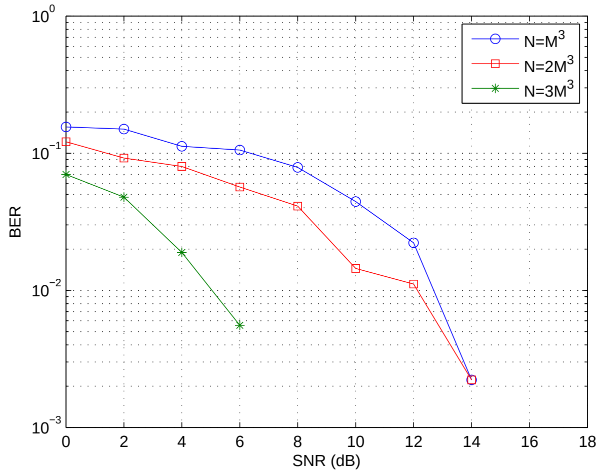

Figure 3: Performance evaluation with different number of rows for the encoding matrix (Joint diagonalization).

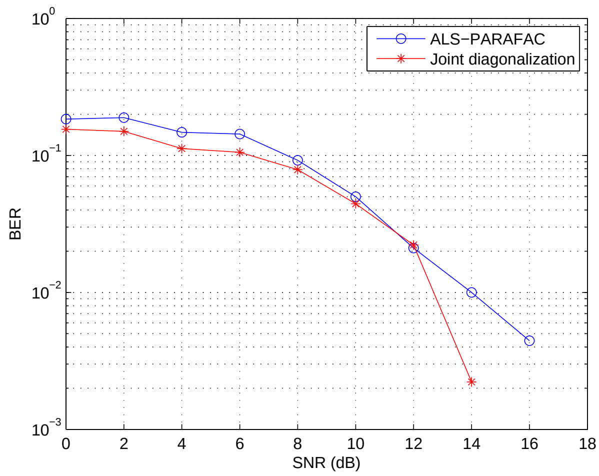

Figure 4: Comparison of the decoding methods ($N = M^3$).

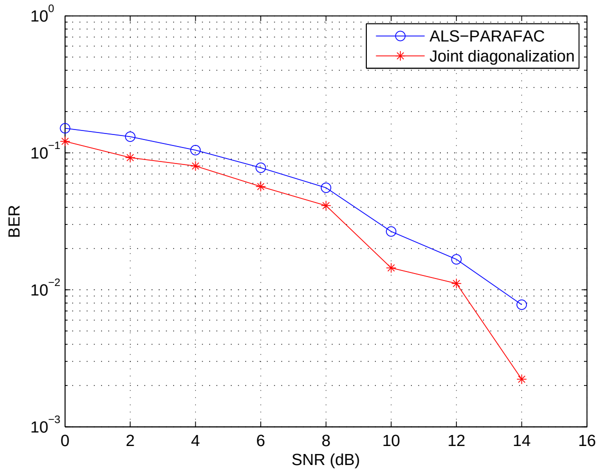

Figure 5: Comparison of the decoding methods ($N = 2M^3$).

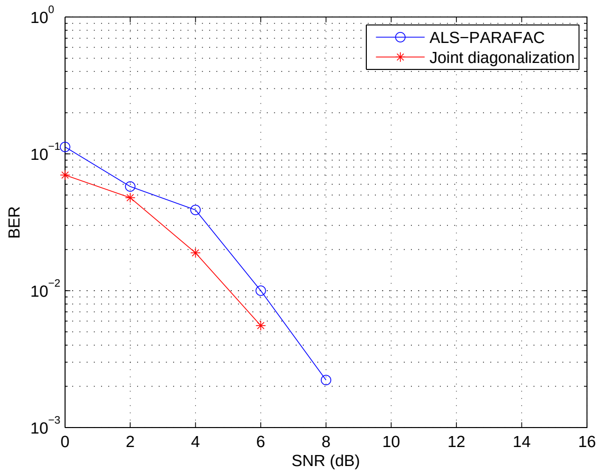

Figure 6: Comparison of the decoding methods ($N = 3M^3$).

5. CONCLUSION

In this paper, we have proposed two blind decoding schemes for multi-input single-output (MISO) communication systems. At the transmitter end, we have introduced a new nonlinear precoding scheme that consists in first linearly precoding the informative symbol with the same matrix for all the users and then nonlinearly mapping the linearly encoded data. By considering a polynomial mapping of degree higher than two, the received signal can be written as the output of an homogeneous Volterra-like model. The input of this model solely depend of the coding sequence assumed to be known to the receiver while the kernel is a multilinear array depending on informative data and on the channel parameters. The proposed decoding scheme is a two-stage one. First, the data kernel is estimated in the least squares sense. Second, the kernel is decomposed using the ALS-PARAFAC method or a joint diagonalization of matrices constructed from the tensor slices. We have shown the efficiency of the proposed methods through simulation results.

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